Box 11.4 – Atmospheric Distribution

We consider a cylinder (cross-section A) filled with a gas (molecules with mass m) as displayed in Figure 11.23. We select a layer of thickness Δx (volume $V = A \cdot \Delta x$). The number N of molecules in this layer is related to the pressure P:

$$PV = NkT, \quad N = \frac{PV}{kT} = \frac{PA \cdot \Delta x}{kT}$$

The force of gravity acting on one molecule is $f = mg$ (g = gravitational acceleration). This means that there is a pressure change $\Delta P = P_{x+\Delta x} - P_x$ in the gas

$$\Delta P = -\frac{N \cdot mg}{A} = -\frac{P\Delta x \cdot mg}{kT}$$

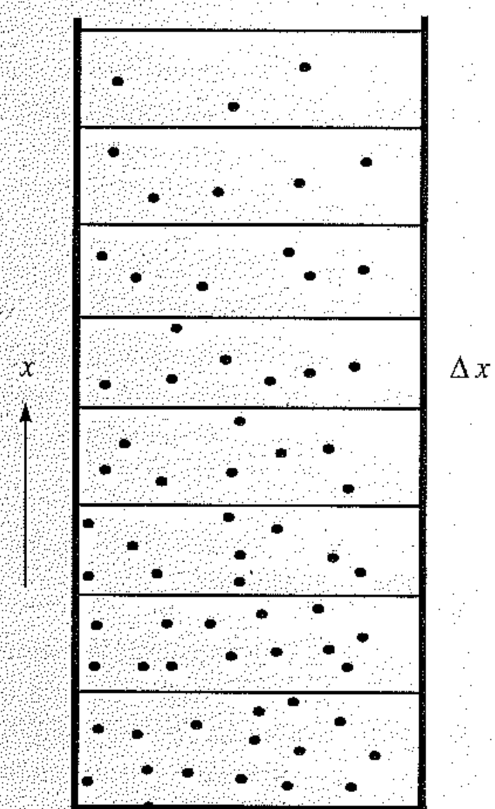

Figure 11.23 Layers of thickness Δx (atmospheric distribution).

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By replacing the differences by differentials

$$dP = -P\frac{mg}{kT} \cdot dx \quad \text{or} \quad \frac{dP}{P} = -\frac{mg}{kT} \cdot dx$$

and integration we obtain the atmospheric distribution function

$$P = P_0 \cdot e^{-mgx/(kT)} \quad \text{or} \quad c = c_0 \cdot e^{-mgx/(kT)}$$

where P_0 is the pressure at $x = 0$ and c_0 is the concentration at $x = 0$.

Example

With $x = 8882\,\text{m}$ (this is the height of Mount Everest), $m = 28\,\text{g mol}^{-1}/(6.022 \times 10^{23}\,\text{mol}^{-1}) = 4.65 \times 10^{-26}\,\text{kg}$ (value for nitrogen) and $g = 9.81\,\text{m/s}^2$ we obtain for $T = 273\,\text{K}$ (as an average of the temperature at $x = 0$ and $x = 8882\,\text{m}$), $P = P_0 \cdot e^{-1.08} = 0.34 \cdot P_0$. This means that the atmospheric pressure on top of Mount Everest is only one-third of the atmospheric pressure on sea level. On the other hand, if we choose x on the order of laboratory dimensions ($x = 1\,\text{m}$) we obtain $P = 0.99987P_0$; this is a decrease by 1×10^{-4} only.
