We'll start by summing the radii of the Rayleigh disk and spherical-aberration disk (in the Gaussian image plane) in quadrature (remember it's radii for image resolution, diameters for probe-limited resolutions)

$$r = (r_{\text{th}}^2 + r_{\text{sph}}^2)^{1/2} \tag{6.19}$$

Therefore, since both these terms are approximate

$$r(\beta) \approx \left[\left(\frac{\lambda}{\beta} \right)^2 + \left(C_{\text{s}} \beta^3 \right)^2 \right]^{1/2} \tag{6.20}$$

Since the two terms vary differently with the aperture collection angle β, a compromise value exists when the differential of $r(\beta)$ with respect to β is set to zero and we find that

$$\frac{\lambda^2}{\beta^3} \approx C_{\text{s}}^2 \beta^5 \tag{6.21}$$

So we come up with an optimum expression for β which Hawkes (1972) gives as

$$\beta_{\mathrm{opt}} = 0.77 \frac{\lambda^{1/4}}{C_s^{1/4}} \tag{6.22}$$

The exact value of the numerical factor depends on the assumptions made about the various terms included in the definition of resolution and so is often written simply as ***A***. Sometimes, this compromise value is determined by simply equating the equations for r_{th} and r_{sph} rather than going through the summation in quadrature. A quick calculation for 100-keV electrons ($\lambda = 0.0037$ nm) for an instrument with $C_s = 3$ mm gives a β_{opt} value of ~4.5 mrads.

If this expression for β_{opt} in equation 6.22 is substituted into equation 6.20 we get a minimum value of $r(\beta)$

$$r_{\mathrm{min}} \approx 0.91 \left(C_s \lambda^3 \right)^{1/4} \tag{6.23}$$

This is the expression we want; it gives the *practical* resolution of the TEM.

The numerical factor in equation 6.23 is often written as ***B***. Typically, the value for r_{min} is ~0.25–0.3 nm and the best high-resolution instruments have r_{min} ~0.1–15 nm; 1-Å TEMs are about the best available without C_s correction and about 0.07 nm is (at the time of writing) the best reported resolution with C_s correction. So, as we showed back in Figure 1.2, we can resolve rows of atoms, which in most crystalline materials have a separation close to r_{min} (although low-index planes in some metals are still below this resolution). It's worth noting that since your eyes can resolve a distance of ~0.2 mm, then the maximum useful magnification of the best high-resolution TEM is ~3×10^6. Above this magnification, no more detail will be revealed.